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Continuous dependence of a coupled system of Wave-Plate Type

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ABSTRACT

In this study, we prove continuous dependence of solutions on coefficients of a coupled system of wave-plate type.

Keywords: Wave-plate type, continuous dependence.

Wave-Plate Tipi denklem sisteminin sürekli bağımlılığı

ÖZ

Bu çalışmada, wave-plate tipi denklem sisteminin çözümlerinin katsayılara sürekli bağımlılığı ispatlanmıştır.

Anahtar Kelimeler: Wave-plate tipi, sürekli bağımlılık.

1. INTRODUCTION

In this paper, we consider the following coupled system of wave-plate type:

$$\alpha u_{tt} - \Delta u - \mu \Delta u_t + a\Delta v = 0, x \in \Omega, \quad t > 0 \qquad (1)$$

$$\beta v_{tt} + \gamma \Delta^2 v + a\Delta u - h\Delta v_t = 0, x \in \Omega, \quad t > 0 \qquad (2)$$

$$(u(x,0), v(x,0)) = (u_0(x), v_0(x)), x \in \Omega, \qquad (3)$$

$$(u_t(x,0), v_t(x,0)) = (u_1(x), v_1(x)), x \in \Omega, \qquad (4)$$

$$u = v = \frac{\partial v}{\partial \nu} = 0, \quad x \in \partial\Omega, t > 0. \qquad (5)$$

Here Ω is a open set of R^n with smooth boundary $\partial\Omega$; $\alpha, \beta, \gamma, \mu$, a and h are positive constants.

Continuous dependence of solutions of problems in partial differential equations on coefficients in the equations is a type of structural stability, which reflects the effect of small changes in coefficient of equations on the solutions. This type has been extensively studied in recent years for a variety of problems. Many results of this type can be found in the literature (see, 1-14, 16, 17, 20-22, 24). Most

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of the paper in the literature study structural stability for various systems in a finite region. For a review of such works, one can refer to [4, 18-20] and papers cited therein. Also, many papers in the literature have studied the Brinkman, Darcy, Forchheimer and Brinkman Forchheimer equations, see [2, 3, 8-16].

In [15], Santos and Munoz Rivera studied the analytic property and the exponential stability of the C_0-semigroup associated with the following coupled system of wave-plate type with thermal effect:

$$\rho_1 u_{tt} - \Delta u - \mu \Delta u_t + \alpha \Delta v = 0, \tag{6}$$

$$\rho_2 v_{tt} + \gamma \Delta^2 v + a \Delta u + m \Delta \theta = 0, \tag{7}$$

$$\tau \theta_t + k \Delta \theta - m \Delta v = 0, \tag{8}$$

where the functions u and v represent the vertical deflections of the membrane and the plate, respectively, θ is the difference between the two temperatures and finally $\rho_1, \rho_2, \mu, \gamma, k, m$ and τ are positive constants. The above model can be used to describe the evolution of a system consisting of an elastic membrane and an elastic plate, subject to a thermal effect and attracting each other by an elastic force with coefficient $\alpha > 0$.

In 2014, Tang, Liu and Liao [23] studied the spatial behavior of the following coupled of the wave-plate type:

$$\rho_1 u_{tt} - \Delta u - \mu \Delta u_t + a \Delta v = 0, \tag{9}$$

$$\rho_2 v_{tt} + \gamma \Delta^2 v + a \Delta u - \frac{m^2}{k} \Delta v_t = 0. \tag{10}$$

The authors got the alternative results of Phragmen-Lindelof type in terms of an area measure of the amplitude in question based on a first-order differential inequality. They also got the spatial decay estimates based on a second-order differential inequality.

Throughout in paper, $\|.\|$ and $(,)$ denote the norm and inner product $L^2(\Omega)$.

2. A PRIORI ESTIMATES

Theorem 1. Let u and v be the solutions of the problem (1)-(5). Then the following estimate holds:

$$\begin{aligned} &\|u_{tt}\|^2 \le D_1(t), \|v_{tt}\|^2 \le D_2(t), \\ &\|\nabla u_t\|^2 \le D_3(t), \|\Delta v_t\|^2 \le D_4(t), \end{aligned} \tag{11}$$

where $D_1(t) = \frac{2}{\alpha} D_0(t)$, $D_2(t) = \frac{2}{\beta} D_0(t)$, $D_3(t) = 2D_0(t)$, $D_4(t) = \frac{2}{\gamma} D_0(t)$, and $D_0(t)$ is a function depending on the initial data and the parameters of (1)-(2).

Proof. Firstly, we differentiate (1) and (2) with respect to t:

$$\alpha u_{ttt} - \Delta u_t - \mu \Delta u_{tt} + a \Delta v_t = 0, \tag{12}$$

and

$$\beta v_{ttt} + \gamma \Delta^2 v_t + a \Delta u_t - h \Delta v_{tt} = 0. \tag{13}$$

Multiplying (12) and (13) by u_{tt} and v_{tt} in $L^2(\Omega)$, respectively we get

$$\begin{aligned} &\frac{d}{dt} E_1(t) + \mu \|\nabla u_{tt}\|^2 + h \|\nabla v_{tt}\|^2 = \\ &a(\nabla u_t, \nabla v_{tt}) + a(\nabla v_t, \nabla \mathrm{u}_{tt}), \end{aligned} \tag{14}$$

where

$$E_1(t) = \frac{\alpha}{2}\|u_{tt}\|^2 + \frac{\beta}{2}\|v_{tt}\|^2 + \frac{1}{2}\|\nabla u_t\|^2 + \frac{\gamma}{2}\|\Delta v_t\|^2.$$

Using the Cauchy's inequality with ε and the Sobolev inequality two terms on the right hand side of (14) we obtain

$$a(\nabla u_t, \nabla v_{tt}) \le \varepsilon_1 \|\nabla v_{tt}\|^2 + \frac{a^2}{4\varepsilon_1}\|\nabla u_t\|^2 \tag{15}$$

and

$$\begin{aligned} a(\nabla v_t, \nabla u_{tt}) &\le \varepsilon_2 \|\nabla u_{tt}\|^2 + \frac{a^2}{4\varepsilon_2}\|\nabla v_t\|^2 \\ &\le \varepsilon_2 \|\nabla u_{tt}\|^2 + \frac{a^2}{4\varepsilon_2} d_1 \|\Delta v_t\|^2, \end{aligned} \tag{16}$$

where d_1 is the positive constant in the Sobolev inequality. From (15) and (16) with ε_1 and ε_2 are selected sufficiently small we obtain

$$\frac{d}{dt} E_1(t) \le M_1 E_1(t), \tag{17}$$

where M_1 is a positive constant depending on the parameters of (1) and (2). So

$$\|u_{tt}\| \le \frac{2}{\alpha} D_0(t), \quad \|v_{tt}\| \le \frac{2}{\beta} D_0(t),$$

$$\|\nabla u_t\| \le 2D_0(t), \quad \|\Delta v_t\| \le \frac{2}{\gamma} D_0(t),$$

where $D_0(t) = E_1(0)e^{M_1 t}$. Therefore (11) is satisfied.

3. CONTINUOUS DEPENDENCE ON PARAMETERS

In this section, we prove that the solution of the problem (1)-(5) depends continuously on μ and h.

Now assume that (u_1, v_1) is the solution of the problem

$$\alpha(u_1)_{tt} - \Delta u_1 - \mu_1 \Delta(u_1)_t + a\Delta v_1 = 0 \quad x \in \Omega, \quad t > 0$$

$$\beta(v_1)_{tt} + \gamma \Delta^2 v_1 + a\Delta u_1 - h\Delta(v_1)_t = 0 \quad x \in \Omega, \quad t > 0$$

$$(u_1(x,0), v_1(x,0)) = (u_0(x), v_0(x)) \quad x \in \Omega,$$

$$((u_1)_t(x,0), (v_1)_t(x,0)) = (u_1(x), v_1(x)) \quad x \in \Omega,$$

$$u_1 = v_1 = \frac{\partial v_1}{\partial \nu} = 0, \quad x \in \partial\Omega, \quad t > 0$$

and (u_2, v_2) is the solution of the following problem

$$\alpha(u_2)_{tt} - \Delta u_2 - \mu_2 \Delta(u_2)_t + a\Delta v_2 = 0 \quad x \in \Omega, \quad t > 0,$$

$$\beta(v_2)_{tt} + \gamma \Delta^2 v_2 + a\Delta u_2 - h\Delta(v_2)_t = 0 \quad x \in \Omega, \quad t > 0,$$

$$(u_2(x,0), v_2(x,0)) = (u_0(x), v_0(x)) \quad x \in \Omega,$$

$$((u_2)_t(x,0), (v_2)_t(x,0)) = (u_2(x), v_2(x)) \quad x \in \Omega,$$

$$u_2 = v_2 = \frac{\partial v_2}{\partial \nu} = 0, \quad x \in \partial\Omega,$$

Let $u = u_1 - u_2$, $v = v_1 - v_2$ and $\mu = \mu_1 - \mu_2$. Then (u, v) satisfies the problem

$$\begin{aligned} &\alpha u_{tt} - \Delta u - \mu_1 \Delta u_t - \mu\Delta(u_2)_t \\ &+ a\Delta v = 0 \quad x \in \Omega, \quad t > 0, \end{aligned} \tag{18}$$

$$\beta v_{tt} + \gamma \Delta^2 v + a\Delta u - h\Delta v_t = 0 \quad x \in \Omega, \quad t > 0, \tag{19}$$

$$(u(x,0), v(x,0)) = (0,0) \quad x \in \Omega, \tag{20}$$

$$(u_t(x,0), v_t(x,0)) = (0,0) \quad x \in \Omega, \tag{21}$$

$$u = v = \frac{\partial v}{\partial \nu} = 0, \quad x \in \partial\Omega, \quad t > 0. \tag{22}$$

Firstly the following theorem establishes continuous dependence of the solution of (1)-(5) on the coefficient μ.

Theorem 2. Let u and v be the solutions of the problem (18)-(22). Then the following estimate holds:

$$\|u_t\|^2 + \|v_t\|^2 + \|\Delta v\|^2 + \|\nabla u\|^2 \le (\mu_1 - \mu_2)^2 A_1(t), \quad \forall t > 0 \tag{23}$$

Proof. Multiplying (18) and (19) by u_t and v_t in $L^2(\Omega)$, respectively and adding the obtained relations, we get

$$\begin{aligned} &\frac{d}{dt} E_2(t) + \mu_1 \|\nabla u_t\|^2 + h\|\nabla v_t\|^2 + \\ &\mu\left(\nabla(u_2)_t, \nabla u_t\right) - a\left(\nabla u_t, \nabla v\right) - \\ &a\left(\nabla v_t, \nabla u\right) = 0, \end{aligned} \tag{24}$$

where

$$E_2(t) = \frac{\alpha}{2}\|u_t\|^2 + \frac{\beta}{2}\|v_t\|^2 + \frac{\gamma}{2}\|\Delta v\|^2 + \frac{1}{2}\|\nabla u\|^2.$$

Using the Cauchy's inequality with ε for sufficiently small $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$, we can write the following inequality:

$$\begin{aligned} &\frac{d}{dt} E_2(t) + (\mu_1 - \varepsilon_1 - \varepsilon_2)\|\nabla u_t\|^2 + (h - \varepsilon_3)\|\nabla v_t\|^2 \le \\ &\frac{\mu^2}{2\varepsilon_1}\left\|\nabla(u_2)_t\right\|^2 + \frac{a^2}{2\varepsilon_2}\|\nabla v\|^2 + \frac{a^2}{4\varepsilon_3}\|\nabla u\|^2. \end{aligned} \tag{25}$$

Then there exist $\mu_1 \ge \varepsilon_1 + \varepsilon_2$ and $h \ge \varepsilon_3$ such that

$$\frac{d}{dt} E_2(t) \le \frac{\mu^2}{2\varepsilon_1}\left\|\nabla(u_2)_t\right\|^2 + \frac{a^2}{2\varepsilon_2}\|\nabla v\|^2 + \frac{a^2}{4\varepsilon_3}\|\nabla u\|^2. \tag{26}$$

So, by using the Sobolev inequality in (25) we find

$$\frac{d}{dt} E_2(t) \le \frac{\mu^2}{2\varepsilon_1}\left\|\nabla(u_2)_t\right\|^2 + \frac{a^2 d_2}{2\varepsilon_2}\|\Delta v\|^2 + \frac{a^2}{4\varepsilon_3}\|\nabla u\|^2, \tag{27}$$

where d_2 is a positive constant in the Sobolev inequality. Inequality (26) implies

$$\frac{d}{dt} E_2(t) \le \frac{\mu^2}{2\varepsilon_1}\left\|\nabla(u_2)_t\right\|^2 + M_2 E_2(t), \tag{28}$$

where $M_2 = a^2 \max\left\{1, \frac{d_2}{\varepsilon_2 \gamma}, \frac{1}{2\varepsilon_3}\right\}$. If we choose

$\varepsilon_1 = \frac{\mu_1}{2}$, then we can write

$$\frac{d}{dt} E_2(t) - M_2 E_2(t) \le \frac{\mu^2}{\mu_1} \left\| \nabla (u_2)_t \right\|^2. \qquad (29)$$

Finally, Gronwall's inequality gives

$$E_2(t) \le \mu^2 A_1(t),$$

where

$$A_1(t) = \frac{1}{\mu_1} e^{M_2 t} \int_0^t \left\| \nabla (u_2)_s \right\|^2 ds.$$

Hence the statement of the theorem holds and we have

$$\|u_t\|^2 + \|v_t\|^2 + \|\Delta v\|^2 + \|\nabla u\|^2 \to 0$$

as $\mu \to 0$.

Finally, we show that the solution of the problem (1)-(5) depends continuously on the coefficient h. Assume that (u_1, v_1) is the solution of the problem

$$\alpha (u_1)_{tt} - \Delta u_1 - \mu \Delta (u_1)_t + a \Delta v_1 = 0 \quad x \in \Omega, \quad t > 0,$$
$$\beta (v_1)_{tt} + \gamma \Delta^2 v_1 + a \Delta u_1 - h_1 \Delta (v_1)_t = 0 \quad x \in \Omega, \quad t > 0,$$

$$(u_1(x,0), v_1(x,0)) = (u_0(x), v_0(x)) \quad x \in \Omega,$$

$$((u_1)_t(x,0), (v_1)_t(x,0)) = (u_1(x), v_1(x)) \quad x \in \Omega,$$

$$u_1 = v_1 = \frac{\partial v_1}{\partial \nu} = 0, \quad x \in \partial\Omega, \quad t > 0,$$

and (u_2, v_2) is the solution of the following problem

$$\alpha (u_2)_{tt} - \Delta u_2 - \mu \Delta (u_2)_t + a \Delta v_2 = 0 \quad x \in \Omega, \quad t > 0,$$

$$\beta (v_2)_{tt} + \gamma \Delta^2 v_2 + a \Delta u_2 - h_2 \Delta (v_2)_t = 0 \quad x \in \Omega, \quad t > 0,$$

$$(u_2(x,0), v_2(x,0)) = (u_0(x), v_0(x)) \quad x \in \Omega,$$

$$((u_2)_t(x,0), (v_2)_t(x,0)) = (u_2(x), v_2(x)) \quad x \in \Omega,$$

$$u_2 = v_2 = \frac{\partial v_2}{\partial \nu} = 0, \quad x \in \partial\Omega, \quad t > 0.$$

Let $u = u_1 - u_2$, $v = v_1 - v_2$ and $h = h_1 - h_2$. Then (u, v) satisfies the problem

$$\alpha u_{tt} - \Delta u - \mu \Delta u_t + a \Delta v = 0 \quad x \in \Omega, \quad t > 0, \qquad (30)$$

$$\beta v_{tt} + \gamma \Delta^2 v + a \Delta u - h_1 \Delta v_t - h \Delta (v_2)_t = 0 \quad x \in \Omega, \quad t > 0, \qquad (31)$$

$$(u(x,0), v(x,0)) = (0,0) \quad x \in \Omega, \qquad (32)$$

$$(u_t(x,0), v_t(x,0)) = (0,0) \quad x \in \Omega, \qquad (33)$$

$$u = v = \frac{\partial v}{\partial \nu} = 0, \quad x \in \partial\Omega, \quad t > 0. \qquad (34)$$

The last result of this section is the following theorem.

Theorem 3. Let u and v be the solutions of the problem (30)-(34). Then the following inequality holds:

$$\|u_t\|^2 + \|v_t\|^2 + \|\Delta v\|^2 + \|\nabla u\|^2 \le (h_1 - h_2)^2 A_2(t), \quad \forall t > 0. \qquad (35)$$

Proof. Multiplying (30) and (31) by u_t and v_t in $L^2(\Omega)$, respectively and adding the obtained relations , we obtain

$$\frac{d}{dt} E_2(t) + \mu \|\nabla u_t\|^2 + h_1 \|\nabla v_t\|^2 + h\left(\nabla (v_2)_t, \nabla v_t\right) +$$

$$a\left(\nabla u_t, \nabla v\right) - a\left(\nabla v_t, \nabla u\right) = 0. \qquad (36)$$

Similar to the proof of Theorem 2, we obtain the following inequality from (36):

$$\frac{d}{dt} E_2(t) \le \frac{h^2}{h_1} \left\| \nabla (v_2)_t \right\|^2 + M_3 E_2(t), \qquad (37)$$

and so, this completes the proof of Theorem 3. Here $E_2(t) = \frac{\alpha}{2}\|u_t\|^2 + \frac{\beta}{2}\|v_t\|^2 + \frac{\gamma}{2}\|\Delta v\|^2 + \frac{1}{2}\|\nabla u\|^2$ and M_3 is a positive constant depending on the parameters of (1)-(2).

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