

Design of fir filters using exponential–Hamming window family

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Abstract: In this paper, a new two-parameter window obtained by the combination of two windows, known as exponential and Hamming, is proposed and applied for the design of finite impulse response (FIR) digital filters. First of all, the quality of the proposed window is analyzed in terms of window spectral parameters and then compared with other two-parameter windows, namely Kaiser, cosh, and exponential. The window simulation results show that the proposed window can provide better ripple ratio than other windows for fixed window length and mainlobe width. Secondly, the performance of the proposed window is analyzed in digital filter design. The filter numerical comparison with Saramaki, Kaiser, Dolph–Chebyshev, cosh, and exponential windows confirms that the filter designed by the proposed window can exhibit a better filtering performance compared to well-known two-parameter windows in the literature because it can provide better minimum stopband attenuation for fixed filter length and transition width.

Key words: FIR digital filter, window function, exponential–Hamming window, exponential window, cosh window, Hamming window, Kaiser window

1. Introduction

Digital signal processing is an area of electrical engineering and applied mathematics that deals with performing useful operations on signals in discrete time. Digital filters can be considered one of the most important and frequently used elements in digital signal processing applications. According to the length of their impulse responses, they are classified as finite impulse response and infinite impulse response (IIR) filters [1].

The popularity of FIR filters is due to the fact that they can be designed as always stable and having a linear phase. On the other hand, as a disadvantage, FIR filters require larger filter lengths (total number of filter coefficients) compared to IIR filters to satisfy the prescribed filter characteristics [1].

In the literature, four general methods, which are Fourier series method using windowing, optimization methods, numerical methods, and frequency sampling method, are used to design FIR digital filters [1]. Among them, first two methods are quite popular in filter design research. Optimum designs using the optimization methods [2] can be obtained, but they require a large amount of computation, which makes them unsuitable for real-time applications [3]. On the other hand, compared to the other methods, the Fourier series method with windowing is the most straightforward one to design FIR filters and involves a minimal amount of computation [3]. The reason to use a window function (or simply window) in the Fourier series method is to truncate and smooth the infinite duration impulse response of the ideal filter [1].

In the literature, many windows for signal spectrum and digital filter design applications have been

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proposed [3–19]. According to the number of independent window parameters in their functions, they can be classified as fixed or adjustable [20]. Rectangular, Von-Hann, Hamming, and Blackman windows [1] are well-known fixed windows. Their functions depend on only the window length. Since they can provide only one characteristic for a fixed filter length, they are not very useful in filter design applications. Although the Hamming window is not attractive for filter design, it is widely used in signal analysis applications. Adjustable windows are attractive in filter design, because for a fixed filter length they can provide many characteristics. Among the adjustable windows, the Kaiser window [5,7] is the best-known and most used window in filter design applications due to its good spectral characteristic and having filter design equations. It is an adjustable window having two independent parameters, namely the window length and the adjustable shape parameter α_k . The exponential window, which was proposed by the author [17,18], is another adjustable window with two parameters. As for comparison with the Kaiser window, it was shown in [18] that although the exponential window is computationally efficient in time due to having no power series expansion in its time domain function, it performs worse filtering in terms of the minimum stopband attenuation for fixed filter length and transition width parameters.

In this paper, a combination of the exponential window and Hamming window is proposed to provide a better filtering performance compared to the Kaiser window. In the next section, the proposed window is introduced with exponential and Hamming windows after a brief explanation about the window theory. In Section 3, the window spectral performance of the proposed window is analyzed and comparisons with Kaiser, cosh [14], and exponential windows are carried out. Then the filters designed by the proposed window are examined and their performances are compared with other well-known two-parameter windows in terms of the minimum stopband attenuation for fixed filter length and transition width parameters in Section 4. The paper is concluded in Section 5.

2. The proposed combinational window: exponential–Hamming window

2.1. Definition and spectral characteristic of a window

A window, denoted as $w(nT)$, is defined as a time function being nonzero for $-n \leq (N-1)/2$ and zero for otherwise [1,14]. In this definition, N is the window length and taken as an odd number, because only windows having odd length and symmetric properties have capability of designing all type of filters [1]. A typical normalized window function in discrete time domain is illustrated in Figure 1.

To be able to give a decision about the quality of a window, it must be analyzed in the frequency domain. The frequency spectrum of an odd length symmetric window can be found by using the following equation [4]:

$$W(e^{jwT}) = |A(w)| e^{j\theta(w)} = w(0) + 2 \sum_{n=1}^{(N-1)/2} w(nT) \cos wnT, \quad (1)$$

where T denotes the sampling period and it is taken as one second ($T = 1$) for the rest of the paper for simplicity. The normalized amplitude spectrum of a window in logarithmic scale can be found from Eq. (2):

$$|W_N(e^{jwT})| = 20 \log_{10}(|A(w)| / |A(w)|_{\max}) \quad (2)$$

The normalized amplitude spectrum of a typical symmetric window in dB range is shown in Figure 2 [4,14]. The main window spectral parameters to distinguish the performances of windows are the mainlobe width (w_M) and ripple ratio I . These parameters are defined as

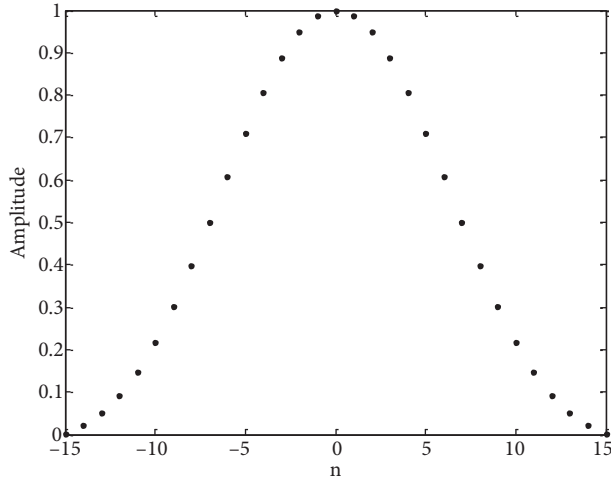


Figure 1. A typical normalized window function plotted for $N = 31$.

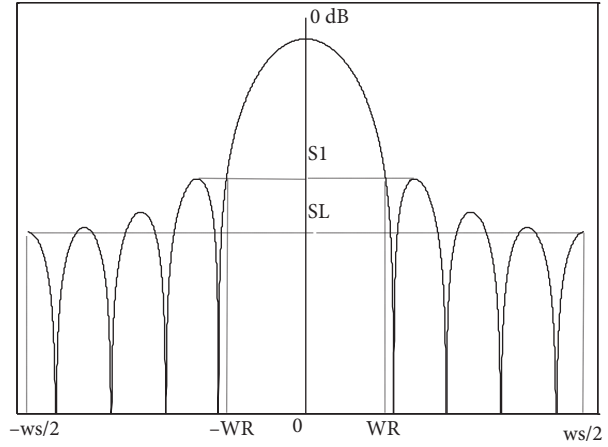


Figure 2. Normalized amplitude spectrum of a typical window.

Mainlobe width: w_M in rad/s = Two times half mainlobe width

Ripple ratio: R in dB = Maximum sidelobe amplitude in dB – Mainlobe amplitude in dB

Therefore, for the spectrum case in Figure 2, these parameters can be easily found as $w_M = 2w_R$ and $R = S_1$.

In the window spectral simulations, half mainlobe width (w_R) and ripple ratio I are used as comparison parameters to be consistent with the literature. In filter design applications, the mainlobe width of a window is directly related to the transition width of a filter. It is required for a good window to have a narrower mainlobe width. The ripple ratio of a window directly affects the ripples in the passband and stopband in a filter. For a good window a smaller ripple ratio is always desired.

3. Exponential window

In discrete time, the two-parameter exponential window is defined in [17,18] as

$$w_e(n) = \frac{e^{\alpha_e \sqrt{1 - \left(\frac{2n}{N-1}\right)^2}}}{e^{\alpha_e}} - n - \leq (N - 1)/2, \quad (3)$$

where N is the window length and α_e is the adjustable independent parameter. This window provides its flexibility due to its two independent spectral parameters, N and α_e .

3.1. Hamming window

The Hamming window, which is widely used in signal processing applications, is defined as [1]

$$w_h(n) = 0.54 - 0.46 \cos \frac{2\pi n}{N-1} \quad n = 0, 1, \dots, N-1 \quad (4)$$

Contrary to the exponential window, it has only one independent parameter, which is the window length N .

3.2. Exponential–Hamming window

To obtain the proposed window, a Hamming window is first shifted in time by 0.5 s, and then added to an exponential window. After that, the addition is scaled by 0.5 to have unity window amplitude. Therefore, the proposed window is defined as

$$w_{eh}(n) = 0.5 \frac{e^{\alpha_{eh} \sqrt{1 - (\frac{2n}{N-1})^2}}}{e^{\alpha_{eh}}} + 0.27 - 0.23 \cos 2\pi(\frac{n}{N-1} + 0.5) \tag{5}$$

Since it is derived from exponential and Hamming windows, it is called the exponential–Hamming window throughout this paper. It is seen from Eq. (5) that it has two independent parameters, namely the window length (N) and the adjustable shape parameter (α_{eh}). Figure 3 shows the time domain characteristic of the proposed window for various values of the parameter α_{eh} with N = 51. It is seen that for larger values of α_{eh} , the exponential–Hamming window has a Gaussian shape.

4. Window spectral analysis of proposed window

4.1. Spectral properties of exponential–Hamming window

In this section the effect of α_{eh} on the proposed window spectrum is observed and then the window spectral relations between the proposed window parameters (N and α_{eh}) and the window spectral parameters (R and w_R) are examined.

The proposed window spectrums for different values of α_{eh} are shown in Figure 4 for a fixed value of length N = 51. Table 1 summarizes Figure 4 as numerical data. It is obviously seen from the figure and table that an increase in α_{eh} results in a wider mainlobe width (becoming worse) and a smaller ripple ratio (becoming better). The results are meaningful because it is contradictory to have a better mainlobe width and ripple ratio at the same time for a two-parameter window. It should be noted from Table 1 that the ripple ratio for $\alpha_{eh} = 9$ is not better than for $\alpha_{eh} = 6$; therefore for α_{eh} values larger than 6 the proposed window is not effective. For detailed investigation about the performance of the proposed window, two window spectral relations mentioned below must be examined for different window lengths.

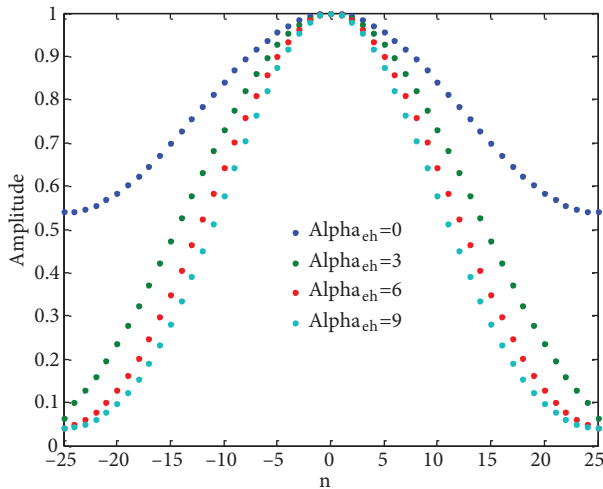


Figure 3. Time domain characteristic of the proposed window for N = 51.

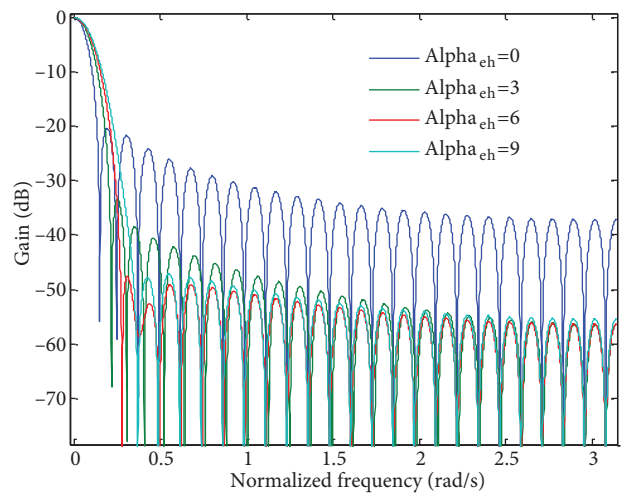


Figure 4. Amplitude spectrum of the proposed window for N = 51.

Table 1. Window spectral values of the proposed window.

Window	N	α_{eh}	w_R (rad/s)	R (dB)
Proposed	51	0	0.1310	-20.32
Proposed	51	3	0.2034	-33.43
Proposed	51	6	0.2673	-47.56
Proposed	51	9	0.3400	-47.15

Figure 5 shows the first window spectral relation between α_{eh} and the ripple ratio for $N = 51$ and 101 . It is obviously seen that the $\alpha_{eh} - R$ relation depends on the window length, while it was shown in [17] that this relation is independent for the exponential window. Therefore, a window spectral design equation satisfying for all N values for this relation cannot be obtained for the proposed window. However, if desired, a design equation for each N value can be obtained by using the curve fitting method in MATLAB. In the same figure, it is also observed that for α_{eh} greater than 6.25 the ripple ratio does not decrease. Therefore, it is better to use the proposed window for α_{eh} less than 6.25.

Figure 6 shows the second window spectral relation between the normalized width parameter (D_w) and the ripple ratio for $N = 51$ and 101 . The normalized width parameter is defined as [4,14,17]

$$D_w = 2w_R(N - 1) \tag{6}$$

Contrary to the case in the exponential window, the window length parameter affects this relation as well, meaning that a window spectral design equation satisfying for all N values for this relation cannot be obtained for the proposed window. In the same figure, it is also observed that for D_w greater than 27.3 rad/s the ripple ratio does not decrease. Therefore, it is better to use the proposed window for D_w less than 27.3 rad/s.

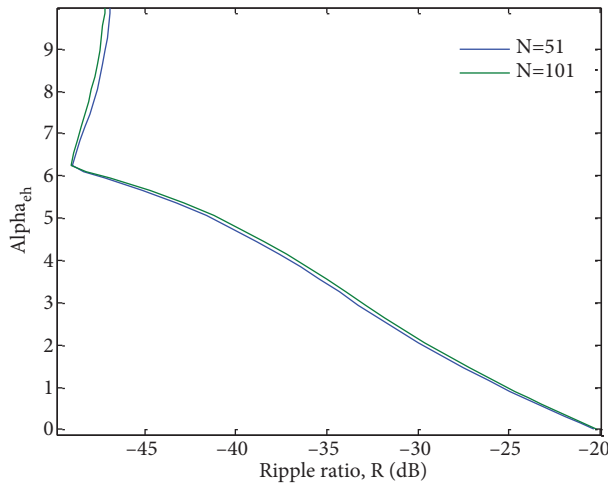


Figure 5. $\alpha_{eh} - R$ relationship of the proposed window for $N = 51$ and 101 .

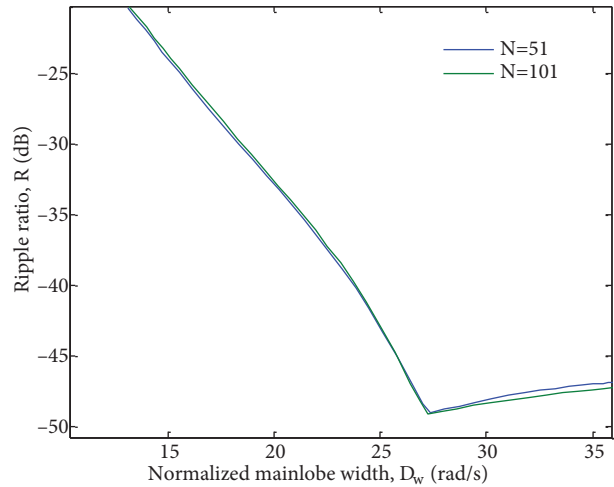


Figure 6. $R - D_w$ relationship of the proposed window for $N = 51$ and 101 .

4.2. Window spectral comparison with Kaiser, cosh, and exponential windows

Figure 7 shows a specific comparison of the proposed window with Kaiser, cosh, and exponential windows for a fixed mainlobe width and window length. It is seen that the best performance in terms of the ripple ratio is provided by the proposed window. The numerical data for the figure is summarized in Table 2.

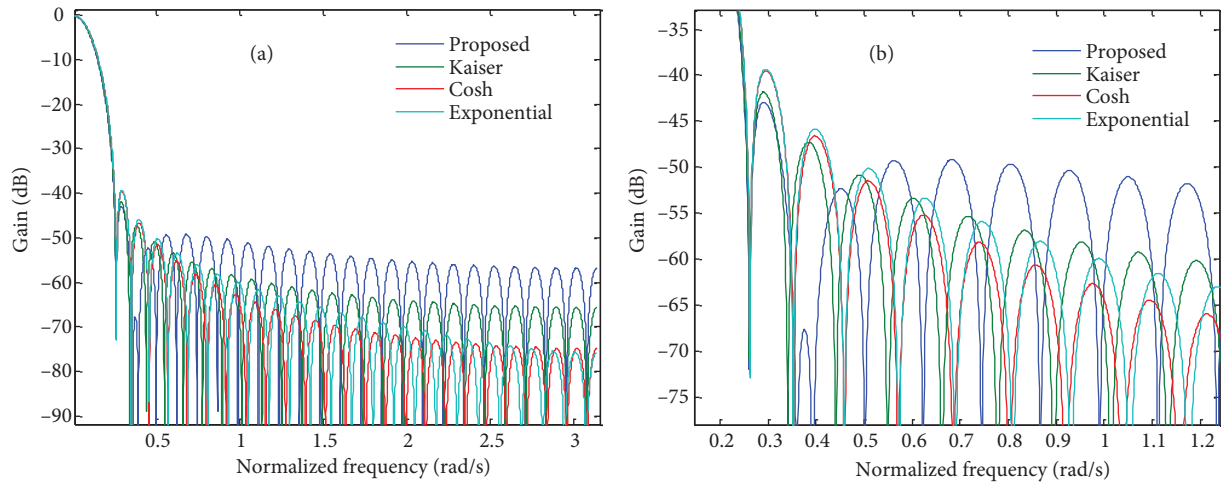


Figure 7. a) Spectral comparison of the proposed window with Kaiser, cosh, and exponential windows for $N = 51$. b) Enlarged figure around the first ripple.

Table 2. Window spectral comparison of the proposed, Kaiser, cosh, and exponential windows.

Window	N	w_R (rad/s)	α	R (dB)
Proposed	51	0.25	5.295	-42.94
Kaiser [5]	51	0.25	5.675	-41.84
Cosh [14]	51	0.25	5.265	-39.51
Exponential [18]	51	0.25	5.290	-39.36

Figure 8 shows a general window spectral comparison of the proposed window with Kaiser, cosh, and exponential windows for $N = 51$ and 101 with a wide range of normalized mainlobe width. Note that Kaiser, cosh, and exponential windows have the same characteristic for $N = 51$ and 101 , but the proposed window has two separate characteristics denoted as $N_{eh} = 51$ and $N_{eh} = 101$ in Figure 8. The figure demonstrates that the proposed window provides better ripple ratio characteristics than the Kaiser window for the range D_w less than 28.5 rad/s, and than the cosh and exponential windows for the range D_w less than 29.7 rad/s.

5. Application of proposed window in FIR digital filter design

5.1. Filter spectral parameters

In this section, the filter spectral parameters mentioned in the filter design examples are introduced. The filter design theory and related examples are given for the lowpass filter type, because other type of filters can easily be designed from it by using suitable transformations. In Figure 9, amplitude spectrum of a lowpass filter is shown with the filter spectral parameters, which are passband frequency (w_p), stopband frequency (w_{st}), maximum passband attenuation (A_p), and minimum stopband attenuation (A_s). The other filter spectral parameters, which are cut-off frequency (w_{ct}) and transition width (Δw), are described as follows:

$$w_{ct} = (w_{st} + w_p)/2 \tag{7}$$

$$\Delta w = w_{st} - w_p \tag{8}$$

It is always desired for a filter to have a narrower Δw , smaller A_p , and greater A_s .

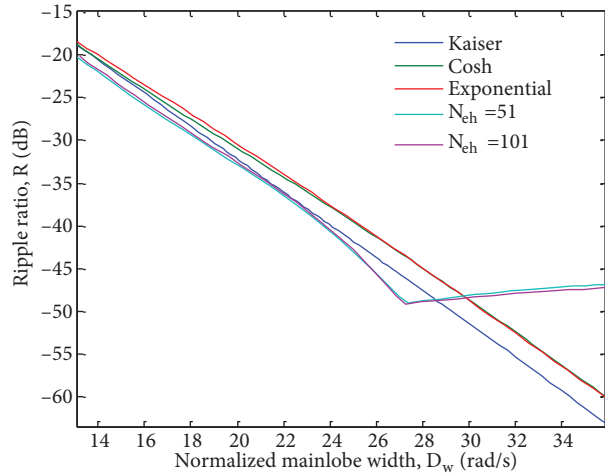


Figure 8. Comparison of the proposed window with Kaiser, cosh, and exponential windows in terms of R and D_w for $N = 51$ and 101 .

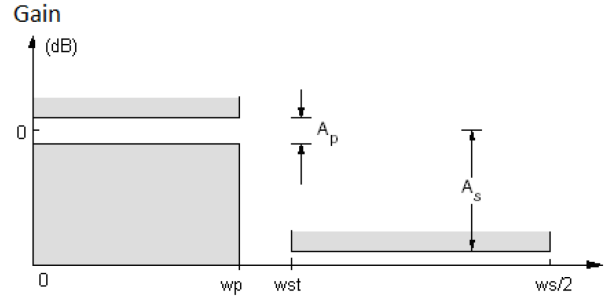


Figure 9. Normalized amplitude spectrum of a lowpass filter.

5.2. FIR digital filter design using windows

To be able to design a FIR filter, it is necessary to find a suitable impulse response that satisfies the prescribed filter specifications. By using a window, $w(nT)$, the impulse response of a realizable noncasual FIR filter can be obtained as [1]

$$h_{nc}(nT) = w(nT)h_{id}(nT), \quad (9)$$

where $h_{id}(nT)$ is the impulse response of the ideal filter with infinite length. For a given cut-off frequency and sampling period (T), it can be found for a low pass filter as [1]

$$h_{id}(nT) = \begin{cases} w_{ct}T/\pi & \text{for } n = 0 \\ \frac{\sin w_{ct}nT}{n\pi} & \text{for } n \neq 0 \end{cases} \quad (10)$$

If the noncasual impulse response $h_{nc}(nT)$ is delayed by a period $(N - 1)/2$, a causal impulse response in order to be realized can be obtained as

$$h_c(nT) = h_{nc}[(n - (N - 1)/2)T] \quad (11)$$

It is known that the ripples in passband and stopband regions of the filters designed by the windowing method are approximately equal to each other [1]. Therefore, only A_s parameter is considered as ripple performance in filter simulation examples.

In order to analyze the designed filter, its amplitude spectrum must be plotted. The frequency spectrum of a filter can be found from its impulse response as

$$H(e^{jwT}) = \sum_{n=0}^{N-1} h(nT)e^{-jwnT} \quad (12)$$

The amplitude spectrum in logarithmic scale can be calculated from

$$|H(e^{jwT})|_{dB} = 20 \log_{10} (|H(e^{jwT})| / |H(e^{jwT})|_{w=0}) \quad (13)$$

5.3. FIR digital filter design using the proposed window

Using Eqs. (5), (9), and (10) lowpass filters can be designed by using the proposed window. The amplitude spectrums of the filters designed for various α_{eh} values at $N = 51$ by using Eq. (13) are shown in Figure 10. The numerical data for this figure are summarized in Table 3. It is seen from the figure and table that as α_{eh} increases the transition width increases (becoming worse) and the minimum stopband attenuation increases (becoming better).

Table 3. Filter spectral values of the lowpass filters designed by the proposed window.

Window	N	α_{eh}	Δw (rad/s)	A_s (dB)
Proposed	51	0	0.2096	32.87
Proposed	51	3	0.3568	48.24
Proposed	51	6	0.4709	58.47
Proposed	51	9	0.5833	64.39

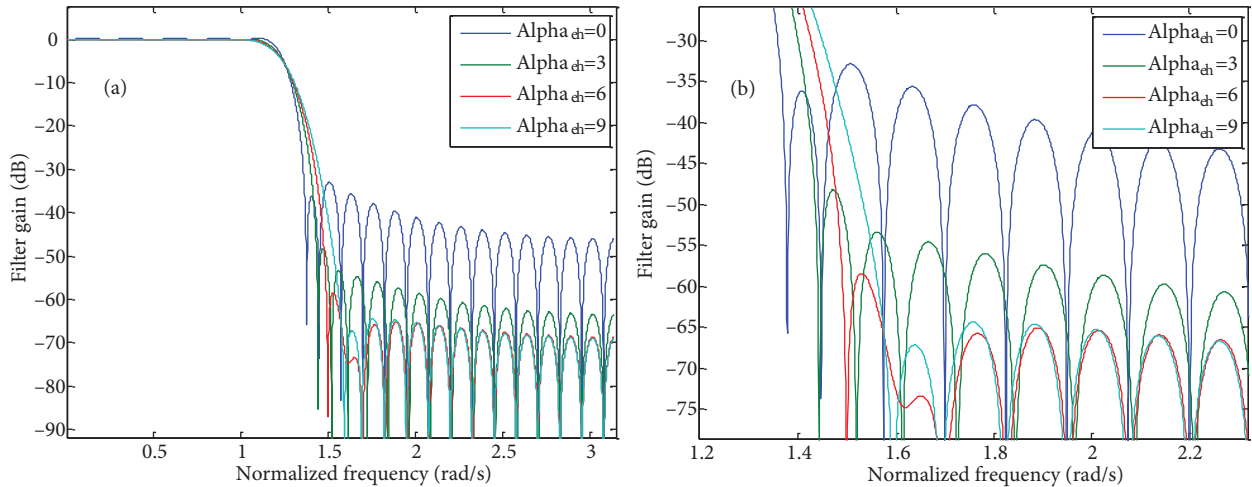


Figure 10. a) Amplitude spectrums of the lowpass filters designed by the proposed window for $N = 51$. b) Enlarged figure around the stopband frequency.

Figure 11 shows the first filter spectral relation between α_{eh} and A_s for the filter lengths $N = 51$ and 127 . It is seen that the $\alpha_{eh} - A_s$ relation depends on the filter length parameter, while it was shown in [18] that this relation is independent of N for the exponential window. Therefore, a filter spectral design equation satisfying for all N values for this relation cannot be obtained for the proposed window. In the same figure, it is also observed that for α_{eh} greater than around 6.9 the minimum stopband attenuation does not increase. Therefore, it is better to use the proposed window for filter applications for α_{eh} less than 6.9.

Figure 12 shows the second filter spectral relation between the normalized transition width parameter and the minimum stopband attenuation for $N = 51$ and 127 . The normalized transition width parameter is defined as [4,14,18]

$$D_f = \Delta w(N - 1)/W_s \tag{14}$$

Contrary to the case in the exponential window [18], the filter length parameter affects the $D_f - A_s$ relation too, meaning that a filter spectral design equation satisfying for all N values for this relation cannot be obtained for the proposed window. In the same figure, it is also observed that for D_f greater than 4.1 rad/s the minimum

stopband attenuation does not increase. Therefore, it is better to use the proposed window for filter applications for D_f less than 4.1 rad/s.

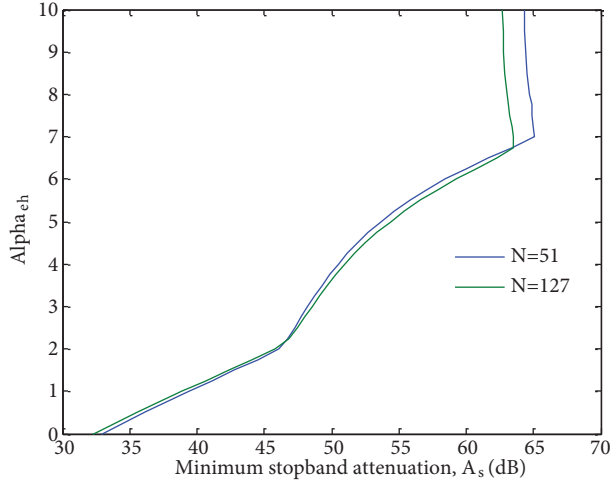


Figure 11. $\alpha_{eh}-A_s$ relationship of the lowpass filters designed by the proposed window.

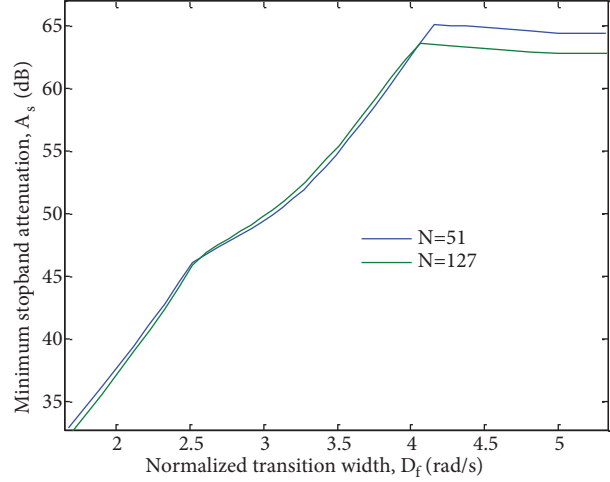


Figure 12. D_f and A_s relationship of the lowpass filters designed by the proposed window.

5.4. Filter comparison examples

In this section, numerical examples are given to demonstrate the performance of the proposed window in filter design. First, a numerical comparison example for the filters designed by the proposed, Saramaki, Kaiser, Dolph–Chebyshev, cosh, and exponential windows is given. The filters are designed to achieve the cut-off frequency $w_{ct} = 0.4\pi$ rad/s and transition width $\Delta w = 0.130$ rad/s for $N = 127$. The comparison result is shown in Figure 13 with summarizing the numerical results for this figure in Table 4. It is obviously seen that the proposed window provides the best filter characteristic, because it provides the largest A_s among all the two-parameter windows. In Table 4, the filter designed by the equiripple optimization method is also included. As expected, the optimization method presents better filtering characteristic than the window method.

Table 4. Filter spectral comparison of lowpass filters designed by proposed and other two-parameter windows for $w_{ct} = 0.4\pi$ rad/s (* Optimization method).

Window	N	Δw (rad/s)	α	A_s (dB)
*Equiripple [2]	127	0.130	-	50.44
Proposed	127	0.130	2.25	46.85
Saramaki [10]	127	0.130	-	45.39
Kaiser [5]	127	0.130	3.97	45.26
Dolph–Chebyshev [1]	127	0.130	-	43.96
Cosh [14]	127	0.130	3.46	43.30
Exponential [18]	127	0.130	3.48	42.70

Figures 14 and 15 show a general filter spectral comparison of the filters designed by the proposed, Kaiser, cosh, exponential, and Hamming windows in terms of the minimum stopband attenuation versus the normalized transition width for $N = 51$ and 127, respectively. It is observed that the filters designed by the proposed window perform better minimum stopband attenuation than the filters designed by the Kaiser window

for D_f less than 2.78 rad/s. As for the comparison with the cosh and exponential windows, the proposed window improved significantly the minimum stopband characteristics for D_f less than 4.28 rad/s and D_f less than 4.29 rad/s, respectively. Since Hamming window has only one independent parameter, it has only one characteristic for a fixed N filter length.

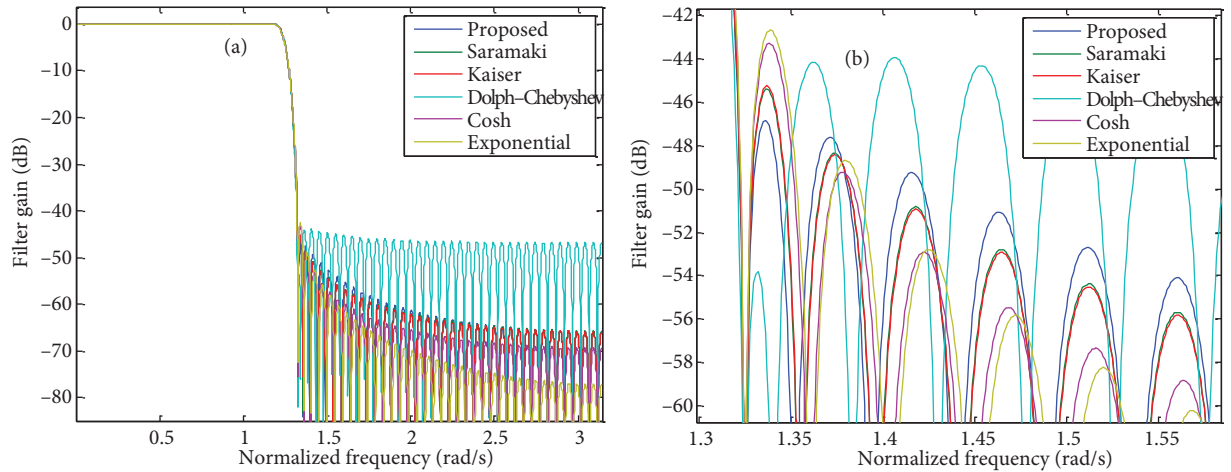


Figure 13. (a) Lowpass filters designed by the proposed and other two-parameter windows for $N = 127$. (b) Enlarged figure around the stopband frequency.

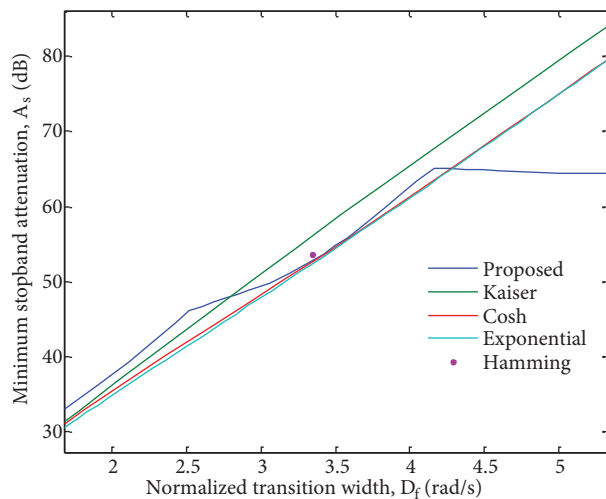


Figure 14. Comparison of lowpass filters designed by the proposed, Kaiser, cosh, exponential, and Hamming windows in terms of A_s and D_f for $N = 51$.

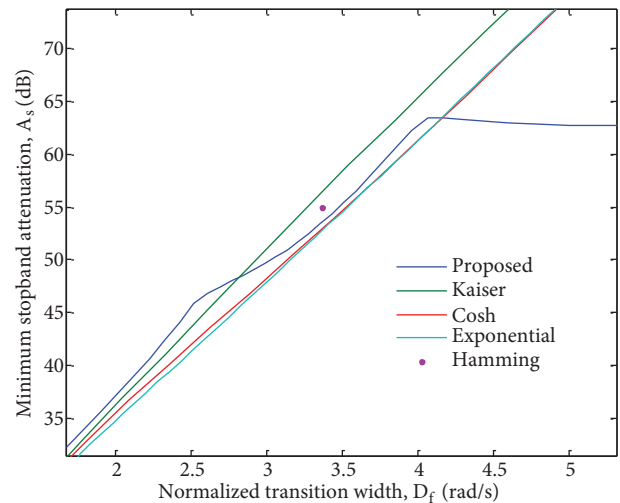


Figure 15. Comparison of lowpass filters designed by the proposed, Kaiser, cosh, exponential, and Hamming windows in terms of A_s and D_f for $N = 127$.

6. Conclusions

In this paper a new window, which is derived from the combination of two windows (namely exponential and Hamming windows) in the literature, is proposed to provide a better filtering performance than the Kaiser window. In FIR filter design applications, the performance of a digital filter is examined with the filter length (N), the minimum stopband attenuation (A_s), and transition width (Δw) parameters. These

filtering parameters have relations with the window spectral parameters window length (N), ripple ratio (R), and mainlobe width ($2w_R$).

In this study, all simulation results are obtained by using MATLAB. In the first part of the study, the quality of the proposed window (namely exponential–Hamming window) is analyzed in terms of window spectral parameters. Then comparisons with two-parameter Kaiser, cosh, and exponential windows are performed. The simulation results for the window spectrum analysis showed that the proposed window provides useful spectral results for α_{eh} less than 6.25. The window comparison results demonstrated that the proposed window provides better ripple ratio characteristics than the Kaiser, cosh, and exponential windows for the ranges D_w less than 28.5, 29.7, and 29.8 rad/s, respectively. Therefore, the proposed window can be useful for some window spectral applications where the Kaiser window is used [7,21–23].

As for the second part of the study, performance of the proposed window in filter design is investigated. For a fair comparison, two of three filtering parameters must be taken as fixed. In this study, the filter length and the transition width parameters are chosen to be fixed; therefore, the filter quality is determined in terms of the minimum stopband attenuation parameter. The simulation results for the filter spectrum analysis showed that the proposed window provides good filtering characteristic for α_{eh} less than 6.9. The filter comparison results showed that the filters designed by the proposed window perform better minimum stopband attenuation than the filters designed by the Kaiser window for D_f less than 2.78 rad/s, and the filters designed by the cosh and exponential windows for D_f less than 2.9 rad/s. As a result, by combining the Hamming window with the exponential window, a better filter characteristic than the Kaiser window can be obtained, whereas neither the exponential window nor the Hamming window can provide better results than the Kaiser window. That is why, instead of the Kaiser window, the proposed window can be used for filters in biomedical and communication applications [24,25].

On the other hand, the proposed window has a disadvantage compared with the Kaiser, cosh, and exponential windows. It has no window design equations [3,14,17] between window parameters (N , α_{eh}) and window spectral parameters (R , w_R) and also has no filter design equations [4,14,18] between window parameters (N , α_{eh}) and filter spectral parameters (A_s , Δw). This is due to the fact that these relations do not remain constant with the parameter N .

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